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http://nptel.ac.in/courses/115101011/1

Galilean transformation

There are the transformations which can transform the coordinates of a particle from one inertial system to another.

Consider two inertial systems F and F′, where F′ is moving with uniform velocity v relative to F along the +ve direction of X-axis (Fig. 11.1). We further consider that the origin of both the systems coincide at time $t = t' = 0$.

Let an event happen point P whose coordinates are (x, y, z, t) and (x', y', z', t') with respect to the frames of references F, and F′ respectively. It can easily be seen that these coordinates are related, as

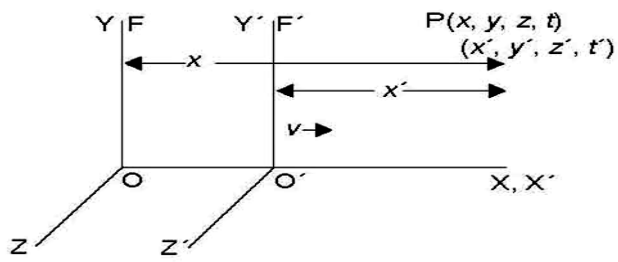
$$\left.\begin{aligned} x' &= x - vt \\ y' &= y \\ z' &= z \\ t' &= t \end{aligned}\right\} \quad \text{(i)}$$

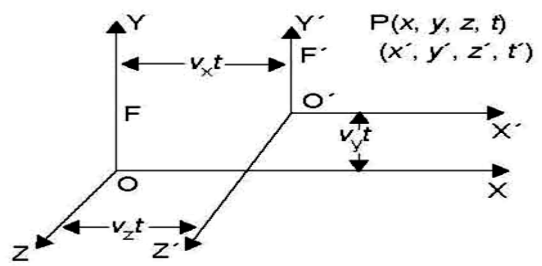
Eq. (i) is known as Galilean transformation for position.

Now, consider that frame F′ is moving along any direction with velocity v relative to F such that

$$\vec{v} = \vec{v}_x \hat{i} + \vec{v}_y \hat{j} + \vec{v}_z \hat{k}$$

where v_x, v_y and v_z are the components of v along X, Y and Z-axes, respectively, as shown in Fig. 11.2. Suppose the origins of the two systems F and F′ coincide at $t = t' = 0$. Let (x, y, z, t) and (x', y', z', t') are the coordinates of the event happening at point P. At the time of event the frame F′ is separated from frame F by a distance $v_x t$, $v_y t$ and $v_z t$ along X, Y and Z-axes respectively. Then we have,





$$\left.\begin{aligned} x' &= x - v_x t \\ y' &= y - v_y t \\ z' &= z - v_z t \\ t' &= t \end{aligned}\right\} \quad \text{(ii)}$$

Eq. (ii) is also known as Galilean transformation for position.

Galilean velocity transformation of the particle can be obtained by differentiating Eq. (ii), with respect to time.

By using $\frac{d}{dt} = \frac{d}{dt'}$ and v_x, v_y and v_z to be constant we obtain

$$\left.\begin{aligned} \frac{dx'}{dt'} &= \frac{dx}{dt} - v_x \\ \frac{dy'}{dt'} &= \frac{dy}{dt} - v_y \\ \frac{dz'}{dt'} &= \frac{dz}{dt} - v_z \end{aligned}\right\} \quad \text{(iii)}$$

$$\text{or} \quad \left.\begin{aligned} u_x' &= u_x - v_x \\ u_y' &= u_y - v_y \\ u_z' &= u_z - v_z \end{aligned}\right\} \quad \text{(iv)}$$

where u_x, u_y and u_z are the velocities of the particle observed by an observer O in system F and u'_x, u'_y and u'_z are the velocities of the particle observed by O′ in system F′ along X, Y and Z-axes, respectively. From Eq. (iv), we have,

$$u_x'\hat{i} + u_y'\hat{j} + u_z'\hat{k} = u_x\hat{i} + u_y\hat{j} + u_z\hat{k} - v_x\hat{i} - v_y\hat{j} - v_z\hat{k}$$

$$\text{or} \quad \vec{u}' = \vec{u} - \vec{v} \quad \text{(v)}$$

where $\hat{i}, \hat{j}, \hat{k}$ are unit vectors along X, Y and Z-axes, respectively. Eq. (v) represents the Galilean transformation of velocity of particle.

Similarly, Galilean acceleration transformation of the particle can be represented by the following equations by knowing the fact that the acceleration of a particle is the time derivative of its velocity. i.e.,

$$a_x = \frac{du_x}{dt}, a_y = \frac{du_y}{dt}, a_z = \frac{du_z}{dt}$$

To find the Galilean acceleration transformations, we differentiate the velocity transformation and use the face that t′ = t and v_x, v_y and v_z are the constants. This yields

$$a_x' = a_x, \; a_y' = a_y \text{ and } a_z' = a_z$$